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Thermal convection in a Kuvshiniski viscoelastic nanofluid saturated porous layer

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Thermophoresis

Abstract The thermal convection in a horizontal layer of a porous medium saturated with a viscoelastic nanofluid was studied analytically. A Kuvshiniski-type constitutive equation is used to describe the behavior of viscoelastic nanofluids. The model used for the viscoelastic nanofluid incorporates the effects of Brownian motion and thermophoresis. A physically more realistic boundary condition than the previous ones on the nanoparticle volume fraction is considered i.e. the nanoparticle flux is assumed to be zero rather than prescribing the nanoparticle volume fraction on the boundaries. Using linear stability theory, the exact analytical expression for the Darcy–Rayleigh number is obtained in terms of various non-dimensional parameters. Results indicate that the coefficient of viscosity, porous medium and nanoparticles significantly influences the stability characteristics of the system. The effect of various parameters on the thermal instability is also presented graphically.

1. Introduction

In current year, huge amount of research and development works have been devoted to nanotechnology. According to Nano Science and Foundation (NSF) the market of nanotechnology will exceed $1 trillion in the USA alone by 2015. About a decade ago Argonne Laboratory started to develop special type of colloid suspension of nanopowders with diameter ranging from 1 nm to 100 nm in host fluids such as water, oil or Ethylene Glycol. Fluids with nanoparticles suspended in them are called nanofluids, a term proposed by Choi [1]. Nanofluids can be considered the next-generation heat transfer fluids as they offer exciting new possibilities to enhance heat transfer performance compared to pure fluids [2]. In recent years, nanofluids are used in automotive radiators, lubrication, additives for fuels, shock absorbers replacing or along with the traditional materials used for similar purposes [3]. The recent articles by Yu and Xie [4], Aybar et al. [5], Sharma et al. [6] and Sheikholeslami et al. [7–10] have covered the latest developments in this field in detail.

A comprehensive survey of convective transport in nanofluids was made by Buongiorno [11]. He noted that the

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Nomenclature

a	dimensionless wave number	μ	viscosity
a_c	critical wave number	$\bar{\mu}$	effective viscosity
C	specific heat	ρ	density of the nanofluid
D	diameter of nanoparticles	λ	coefficient of viscoelasticity
D_B	Brownian diffusion coefficient	ρ_0	reference density of nanofluid
D_T	thermophoretic diffusion coefficient	ρ_p	density of nanoparticles
$\vec{\mathbf{g}}$	acceleration due to gravity	(ρc)	heat capacity
k	thermal conductivity	ϕ	volume fraction of the nanoparticles
L_e	Lewis number	ϕ_0^*	reference scale for the nanoparticle fraction
N_A	modified diffusivity ratio	∇_P^2	horizontal Laplacian operator
N_B	modified specific heat increment	∇^2	Laplacian operator
p	pressure		
P_r	Prandtl number	*Superscripts*	
$\vec{\mathbf{v}}_D$	Darcy velocity of nanofluid	*	dimensional variables
R_D	Darcy–Rayleigh number	$'$	perturbed quantities
$R_{D,c}$	critical Darcy–Rayleigh number		
t	time	*Subscripts*	
T	temperature	p	particle
(x, y, z)	space coordinates	b	basic state
		0	lower boundary
Greek symbols		1	upper boundary
β	coefficient of thermal expansion		

nanoparticles absolute velocity can be viewed as the sum of the base fluid velocity and a relative (slip) velocity. On the basis of this principle, Tzou [12], Nield and Kuznetsov [13–15], Yadav et al. [16–27], Chand and Rana [28–30] studied the problem of thermal instability in nanofluid. The common finding of these studies was that the critical Rayleigh number can be reduced or increased by a substantial amount, depending on whether the basic nanoparticle distribution is top-heavy or bottom-heavy, by the presence of nanoparticles. In studying these convective instability problems, the volume fraction of nanoparticles was prescribed at the boundaries. Two phase simulation of nanofluid flow and heat transfer was studied by Sheikholeslami et al. [31–33]. Recently, Nield and Kuznetsov [34], Yadav et al. [35–39], Shivakumara and Dhananjaya [40] and Chand and Rana [41,42] pointed out that this type of boundary condition on volume fraction of nanoparticles is physically not realistic as it is difficult to control the nanoparticle volume fraction on the boundaries and suggested the normal flux of volume fraction of nanoparticles is zero on the boundaries as an alternative boundary condition which is physically more realistic.

In actual circumstances in nanofluids, the base fluid does not satisfy the properties of Newtonian fluids; hence, it is more justified to consider them as viscoelastic fluids; for example, ethylene glycol–Al_2O_3, ethylene glycol–CuO, and ethylene glycol–ZnO are some examples of viscoelastic nanofluids. With the growing importance of viscoelastic nanofluids in technology and industries, the investigations of such fluids are desirable. Nonetheless, the study of thermal instability in a viscoelastic nanofluid saturated porous layer is relatively of recent origin and it is still in a rudimentary stage. Sheu [43], Yadav et al. [44], Umavathi et al. [45] and Shivakumara et al. [46] studied the onset of convection in a horizontal layer of porous medium saturated by a viscoelastic nanofluid using Oldroyd-B model while Rana and Chand [47] extended this study with Rivlin–Ericksen elastico-viscous model. Thermal instability of a nonhomogeneous power-law nanofluid in a porous layer with horizontal throughflow was studied by Kang et al. [48]. They obtained that the critical Rayleigh number can be significantly reduced or increased with the increasing power-law index, mainly depending on the value of Péclet number.

In the present paper, the base fluid is taken as (Kuvshiniski-type) viscoelastic fluid. To the best of our knowledge, no studies have far been investigated to analyze the onset of thermal convection of (Kuvshiniski-type) viscoelastic nanofluid layer in porous medium. The objective of the present paper was therefore to extend the previous work by taking base fluid as (Kuvshiniski-type) viscoelastic fluid considering the flux of volume fraction of nanoparticles is zero at the boundaries. The effects of the embedded flow controlling parameters on the thermal convection have been demonstrated graphically and discussed.

2. Mathematical formulation of the problem

We consider an infinite horizontal layer of incompressible nanofluid heated from below as shown in Fig. 1. A Cartesian coordinate system (x, y, z) is chosen in which z axis is taken at right angle to the boundaries. The nanofluid is confined between two parallel plates $z^* = 0$ and $z^* = L$, where the temperatures at the lower and upper boundaries are taken as T_0^* and T_1^*, respectively, T_0^* being greater than T_1^*. Asterisks are used to distinguish the dimensional variables from the non-dimensional variables (without asterisks).

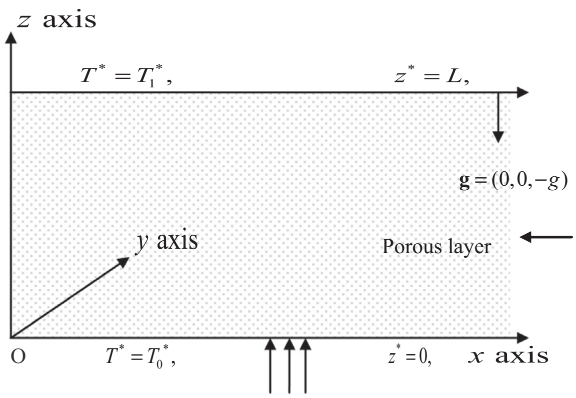

Figure 1 Physical model and coordinate system.

2.1. Assumptions

The mathematical equations describing the physical model are based upon the following assumptions [11]:

(i) the volumetric fraction of nanoparticles is very small (less than 4%);
(ii) the thermophysical properties except for density in the buoyancy force (Boussinesq Hypothesis) are constant;
(iii) the fluid phase and nanoparticles are in thermal equilibrium state and thus, the heat flow has been described using one equation model;
(iv) nanofluid is incompressible and laminar;
(v) each boundary wall is assumed to be impermeable and perfectly thermal conducting;
(vi) radiative heat transfer between the sides of wall is negligible when compared with other modes of the heat transfer.

2.2. Governing equations

The continuity equation for the nanofluid is

$$\nabla^* \cdot \vec{\mathbf{v}}^*_{\mathbf{D}} = 0, \tag{1}$$

where $\vec{\mathbf{v}}^*_{\mathbf{D}} = (u^*, v^*, w^*)$ is the nanofluid Darcy velocity.

If one introduces a buoyancy force and adopts the Boussinesq approximation, then the momentum equation for Kuvshiniski viscoelastic nanofluid can be written as [49,50]

$$\left(1 + \lambda \frac{\partial}{\partial t^*}\right)\left[\frac{\rho_0}{\varepsilon}\frac{\partial \vec{\mathbf{v}}^*_{\mathbf{D}}}{\partial t^*} - \left(\bar{\mu}\nabla^{*^2}\vec{\mathbf{v}}^*_{\mathbf{D}} - \frac{\mu}{K}\vec{\mathbf{v}}^*_{\mathbf{D}}\right)\right]$$
$$= -\nabla^* p^* + [\phi^* \rho_p + \rho_0(1 - \phi^*)\{1 - \beta(T^* - T^*_0)\}]\vec{\mathbf{g}}, \tag{2}$$

where ρ_0 is the density of the nanofluid at the reference temperature T^*_0, ε is the porosity of the porous medium, K is the permeability of the porous medium, β is the thermal expansion coefficient, t^* is the time, p^* is the pressure, ϕ^* is volumetric fraction of nanoparticle, ρ_p is the density of nanoparticle, μ, ρ and λ are the viscosity, density and coefficient of viscoelasticity of nanofluid, respectively and $\bar{\mu}$ is the effective viscosity. It has been generally accepted that $\bar{\mu}$ is strongly dependent on the type of porous medium as well as the strength of flow. Depending upon the type of porous media, the effective viscosity may be either smaller or greater than the viscosity of the nanofluid.

The energy equation is

$$(\rho c)_m \frac{\partial T^*}{\partial t^*} + (\rho c)(\vec{\mathbf{v}}^*_{\mathbf{D}} \cdot \nabla^*)T^* = k_m \nabla^{*^2} T^*$$
$$+ \varepsilon(\rho c)_p \left(D_B \nabla^* \phi^* \cdot \nabla^* T^* + \frac{D_T}{T^*_0}\nabla^* T^* \cdot \nabla^* T^*\right), \tag{3}$$

where ρ is the density of nanofluid, $(\rho c)_m$ is the effective heat capacity, k_m is the effective thermal conductivity, D_B is the Brownian diffusion coefficient and D_T is the thermophoretic diffusion coefficient of the nanoparticles, and (ρc) and $(\rho c)_p$ are the heat capacity of the nanofluid and nanoparticles, respectively. The second term of the right hand side is the additional flow work due to Brownian motion and thermophoresis of nanoparticles relative to the flow velocities.

The conservation equation for the nanoparticles is

$$\frac{\partial \phi^*}{\partial t^*} + \frac{1}{\varepsilon}(\vec{\mathbf{v}}^*_{\mathbf{D}} \cdot \nabla^*)\phi^* = D_B \nabla^{*^2}\phi^* + \frac{D_T}{T^*_0}\nabla^{*^2} T^*, \tag{4}$$

In previous investigations of convective instability problems for nanofluids, the volumetric fraction of nanoparticles was prescribed at the boundaries. But it is observed that this type of boundary condition on volume fraction of nanoparticles is physically not realistic because in practice controlling the nanoparticle volume fraction on the boundaries may be difficult. Thus it is advisable to replace the boundary conditions by a set that are more realistic physically. In this paper, it is assumed that the nanoparticle fraction adjusts so that the nanoparticle flux is zero on the boundaries. This boundary condition on the nanoparticle volume fraction is made possible by accounting for the contributions of the effect of thermophoresis to the nanoparticle flux. In this respect this model is more realistic physically than previous. Thus the boundary conditions are as follows:

$$w^* = 0, T^* = T^*_0, D_B \frac{\partial \phi^*}{\partial z^*} + \frac{D_T}{T^*_0}\frac{\partial T^*}{\partial z^*} = 0 \quad \text{at } z^* = 0, \tag{5a}$$

$$w^* = 0, T^* = T^*_1, D_B \frac{\partial \phi^*}{\partial z^*} + \frac{D_T}{T^*_0}\frac{\partial T^*}{\partial z^*} = 0 \quad \text{at } z^* = L, \tag{5b}$$

Introducing the following non dimensional parameters:

$$(x, y, z) = \frac{(x^*, y^*, z^*)}{L}, t = \frac{t * b\alpha_m}{\sigma L^2},$$
$$(u, v, w) = \frac{(u^*, v^*, w^*)L}{\alpha_m}, p = \frac{p^* K}{\mu \alpha_m}, \phi = \frac{\phi^* - \phi^*_0}{\phi^*_0},$$
$$T = \frac{T^* - T^*_1}{T^*_0 - T^*_1}, \tag{6}$$

where $\alpha_m = \frac{k_m}{(\rho c)}$, $\sigma = \frac{(\rho c)_m}{(\rho c)}$ and ϕ^*_0 is a reference scale for the nanoparticle fraction.

Then, the non-dimensional form of Eqs. (1)–(5) is as follows:

$$\nabla \cdot \vec{\mathbf{v}} = 0, \tag{7}$$

$$\left(1 + \alpha \frac{\partial}{\partial t}\right)\left[\frac{D_a}{P_r}\frac{\partial}{\partial t} - (D_a \nabla^2 - 1)\right]\vec{\mathbf{v}} = -\nabla p - R_m \hat{\mathbf{e}}_z$$
$$+ R_D T \hat{\mathbf{e}}_z - R_n \phi \hat{\mathbf{e}}_z, \tag{8}$$

$$\frac{\partial T}{\partial t} + (\vec{\mathbf{v}} \cdot \nabla)T = \nabla^2 T + \frac{N_B}{L_e}\nabla\phi \cdot \nabla T + \frac{N_A N_B}{L_e}\nabla T \cdot \nabla T, \tag{9}$$

$$\frac{1}{\sigma}\frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon}(\vec{\mathbf{v}} \cdot \nabla)\phi = \frac{1}{L_e}\nabla^2\phi + \frac{N_A}{L_e}\nabla^2 T, \tag{10}$$

$$w = 0, T = 1, \frac{d\phi}{dz} + N_A \frac{dT}{dz} = 0, \quad \text{at } z = 0, \tag{11a}$$

$$w = 0, T = 0, \frac{d\phi}{dz} + N_A \frac{dT}{dz} = 0, \quad \text{at } z = 1. \tag{11b}$$

Here $R_D = \frac{\rho_0 g \beta K L (T_0^* - T_1^*)}{\mu \alpha_m}$ is the Darcy–Rayleigh number, $D_a = \frac{\bar{\mu} K}{\mu L^2}$ is the Darcy number, $L_e = \frac{\alpha_m}{D_B}$ is the Lewis number, $P_r = \frac{\mu}{\rho_0 \alpha_m}$ is the Prandtl number, $N_B = \frac{\varepsilon (\rho c)_p \phi_0^*}{(\rho c)}$ is the modified particle density increment, $N_A = \frac{D_T (T_0^* - T_1^*)}{D_B T_0^* \phi_0^*}$ is the modified diffusivity ratio, $\alpha = \lambda \frac{\alpha_m}{\sigma L^2}$ is the viscoelastic parameter, $R_m = \frac{[\rho_p \phi_0^* + \rho_0 (1 - \phi_0^*)] g K L}{\mu \alpha_m}$ is the basic density Darcy–Rayleigh number, and $R_n = \frac{(\rho_p - \rho_0) \phi_0^* g K H}{\mu \alpha_m}$ is the concentration Darcy–Rayleigh number.

2.3. Basic state

The basic state of the nanofluid is assumed to be time independent and is described by $\mathbf{v} = 0$, $T = T_b(z)$, $p = p_b(z)$, $\phi = \phi_b(z)$.

Then Eqs. (9) and (10) become:

$$\frac{d^2 T_b}{dz^2} + \frac{N_B}{L_n} \frac{dT_b}{dz} \left[\frac{d\phi_b}{dz} + N_A \left(\frac{dT_b}{dz} \right) \right], \tag{12}$$

$$\frac{d^2 \phi_b}{dz^2} + N_A \frac{d^2 T_b}{dz^2} = 0, \tag{13}$$

under the following boundary conditions:

$$T_b = 1, \frac{d\phi_b}{dz} + N_A \frac{dT_b}{dz} = 0 \quad \text{at } z = 0, \tag{14a}$$

$$T_b = 0, \frac{d\phi_b}{dz} + N_A \frac{dT_b}{dz} = 0 \quad \text{at } z = 1. \tag{14b}$$

On solving Eqs. (12) and (13) subject to the boundary conditions (14), we found that

$$T_b(z) = 1 - z \text{ and } \phi_b(z) = N_A z + \phi_0^*, \tag{15a, b}$$

where ϕ_0^* is the reference value of the volumetric fraction of nanoparticle.

2.4. Perturbation equations

For small disturbances onto the primary flow, we assume that:

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}', T = T_b(z) + T', p = p_b(z) + p', \phi = \phi_b(z) + \phi', \tag{16}$$

where prime indicates perturbation quantities over their equilibrium counterparts and assumed to be small. On substituting Eq. (16) into Eqs. (7)–(10) and neglecting the product of prime quantities, we have:

$$\nabla \cdot \vec{\mathbf{v}}' = 0, \tag{17}$$

$$\left(1 + \alpha \frac{\partial}{\partial t}\right) \left[\frac{D_a}{P_r} \frac{\partial}{\partial t} - (D_a \nabla^2 - 1) \right] \vec{\mathbf{v}}' = -\nabla p' + R_D T' \hat{\mathbf{e}}_z - R_n \phi' \hat{\mathbf{e}}_z, \tag{18}$$

$$\frac{\partial T'}{\partial t} - w' = \nabla^2 T' - \frac{N_B}{L_e} \left(N_A \frac{\partial T'}{\partial z} + \frac{\partial \phi'}{\partial z} \right), \tag{19}$$

$$\frac{1}{\sigma} \frac{\partial \phi'}{\partial t} + \frac{N_A}{\varepsilon} w' = \frac{1}{L_e} \nabla^2 \phi' + \frac{N_A}{L_e} \nabla^2 T', \tag{20}$$

Operating on Eq. (18) with $\hat{\mathbf{e}}_\mathbf{z} \cdot (\nabla \times \nabla \times)$ and using the identity curl curl $\equiv$ grad div $- \nabla^2$ together with Eqs. (17), we obtain z-component of the momentum equation as follows:

$$\left(1 + \alpha \frac{\partial}{\partial t}\right) \left[D_a \nabla^2 - \frac{D_a}{P_r} \frac{\partial}{\partial t} - 1 \right] \nabla^2 w' + R_D \nabla_p^2 T' - R_n \nabla_p^2 \phi' = 0, \tag{21}$$

where ∇_p^2 is the Laplacian operator in the horizontal plane.

Assume that the perturbation quantities are of the form as follows:

$$(w', T', \phi') = [W(z), \Theta(z), \Phi(z)] \exp[i k_x x + i k_y y + \omega t], \tag{22}$$

where k_x, k_y are the wave numbers along the x and y directions, respectively and ω is the growth rate.

On substituting Eq. (22) into the differential Eqs. (19)–(21), the linearized equations in dimensionless form are as follows:

$$W + \left(D^2 - a^2 - \omega - \frac{N_A N_B}{L_e} D \right) \Theta - \frac{N_B}{L_e} D \Phi = 0, \tag{23}$$

$$\left[\frac{1}{L_e} (D^2 - a^2) - \frac{\omega}{\sigma} \right] \Phi + \frac{N_A}{L_e} (D^2 - a^2) \Theta - \frac{N_A}{\varepsilon} W = 0, \tag{24}$$

$$(1 + \alpha \omega) \left[D_a (D^2 - a^2) - \frac{D_a}{P_r} \omega - 1 \right] (D^2 - a^2) W - R_D a^2 \Theta + R_n a^2 \Phi = 0, \tag{25}$$

where $D \equiv \frac{d}{dz}$ and $a = \sqrt{k_x^2 + k_y^2}$ are the resultant wave numbers. The boundary conditions become:

$$W = \Theta = 0, \ D\Phi + N_A D\Theta = 0 \quad \text{at } z = 0, 1. \tag{26}$$

Here the growth rate ω is in general a complex quantity such that $\omega = \omega_r + i\omega_i$ the system with $\omega_r < 0$ is always stable, while for $\omega_r > 0$, it will become unstable. For neutral stability, the real part of ω is zero. Hence, we now write $\omega = i\omega_i$, where ω_i is real and is a dimensionless frequency.

Eqs. (23)–(25) together with the boundary conditions (26) constitute a linear eigenvalue problem of the system. The resulting eigenvalue problem is solved analytically using the Galerkin weighted residuals method. In this method, the test (weighted) functions are the same as the base (trial) functions. Accordingly W, Θ and Φ are taken in the following way:

$$W = \sum_{s=1}^{N} A_s W_s, \quad \Theta = \sum_{s=1}^{N} B_s \Theta_s \quad \text{and} \quad \Phi = \sum_{s=1}^{N} C_s \Phi_s, \tag{27}$$

where A_s, B_s and C_s are constants. The base functions W_s, Θ_s and Φ_s are represented by power series as trivial functions satisfying the respective boundary conditions and are assumed in the following form:

$$W_s = \Theta_s = \sin s\pi z, \quad \Phi_s = -N_A \sin s\pi z, \quad s = 1, 2, 3, \ldots, N. \tag{28}$$

Using Eq. (27) into Eqs. (23)–(25) and multiplying Eq. (23) by Θ_s, Eq. (24) by Φ_s and Eq. (25) by W_s; performing the integration by parts with respect to z between $z = 0$ and 1, we obtain a system of $3s$ linear algebraic equations in the $3s$ unknowns A_s, B_s and C_s. For the existence of non trivial solution, the determinant of the coefficient matrix must vanish, which gives the characteristic equation for the system, with Darcy–Rayleigh number R_D as the eigenvalue of the characteristic equation. For a first approximation, we take $s = 1$, and this gives the following expression for the Darcy-Rayleigh number R_D

$$R_D = \Delta_1 + i\omega_i \Delta_2, \tag{29}$$

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where

$$\Delta_1 = \frac{-D_a J(1+J\alpha)\omega_i^2 + J(1+D_a J)(J-\alpha\omega_i^2)P_r}{a^2 P_r} - \frac{N_A R_n \sigma\{L_e^2\omega_i^2 + J^2(\varepsilon+L_e)\sigma\}}{\varepsilon(L_e^2\omega_i^2 + J^2\sigma^2)},$$

$$\Delta_2 = \frac{J[D_a(J-\alpha\omega_i^2) + P_r + J(D_a+\alpha+D_a\alpha\lambda)P_r]}{a^2 P_r} + \frac{J L_e N_A R_n(\varepsilon+L_e-\sigma)\sigma}{\varepsilon(L_e^2\omega_i^2 + J^2\sigma^2)}.$$

Here $J = a^2+\pi^2$. Since R_D is a physical quantity, it must be real values. Hence, it follows from Eq. (29) that either $\omega_i = 0$ (stationary convection) or $\Delta_2 = 0(\omega_i \neq 0$, oscillatory convection).

3. Results and discussion

3.1. Stationary convection

First, consider the case of stationary convection i.e. $\omega_i = 0$. Then, Eq. (29) gives the following expression for the Darcy–Rayleigh number:

$$R_{D,s} = \frac{(a^2+\pi^2)^2[1+D_a(a^2+\pi^2)]}{a^2} - \frac{N_A R_n(\varepsilon+L_e)}{\varepsilon}. \quad (30)$$

From Eq. (30), it is clear that the Darcy–Rayleigh number $R_{D,s}$ increases with increasing the Darcy number D_a and porosity parameter ε while decreases with increasing the concentration Darcy–Rayleigh number R_n, the Lewis number L_e and the modified diffusivity ratio N_A. Also, it is clear from Eq. (30) that the viscoelastic parameter α has no effect on the stationary convection of nanofluid i.e. for the case of stationary convection, the result is same as those of Newtonian nanofluid.

For the case $D_a = 0$, Eq. (30) gives

$$R_{D,s} = \frac{(a^2+\pi^2)^2}{a^2} - \frac{N_A R_n(\varepsilon+L_e)}{\varepsilon}. \quad (31)$$

This result is identical with the result of Yadav et al. [37]. Also, for the case of $D_a = 0$ and in the absence of nanoparticles, Eq. (30) coincides with the result of Kang et al. [51].

3.2. Oscillatory convection

For oscillatory convection $\omega_i \neq 0$, thus we must have $\Delta_2 = 0$, which gives the following expression for the Darcy–Rayleigh number:

$$R_D = \frac{-D_a J(1+J\alpha)\omega_i^2 + J(1+D_a J)(J-\alpha\omega_i^2)P_r}{a^2 P_r} - \frac{N_A R_n \sigma\{L_e^2\omega_i^2 + J^2(\varepsilon+L_e)\sigma\}}{\varepsilon(L_e^2\omega_i^2 + J^2\sigma^2)}, \quad (32)$$

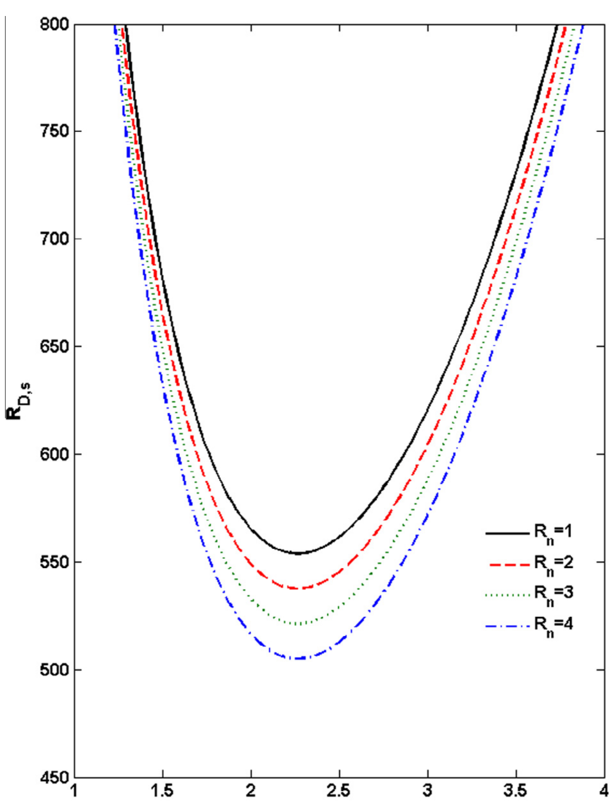

Figure 2 Variation of the Darcy–Rayleigh number $R_{D,s}$ with the wave number a for various values of the concentration Darcy-Rayleigh number R_n with $N_A = 2, L_e = 5, \varepsilon = 0.7, D_a = 0.8$.

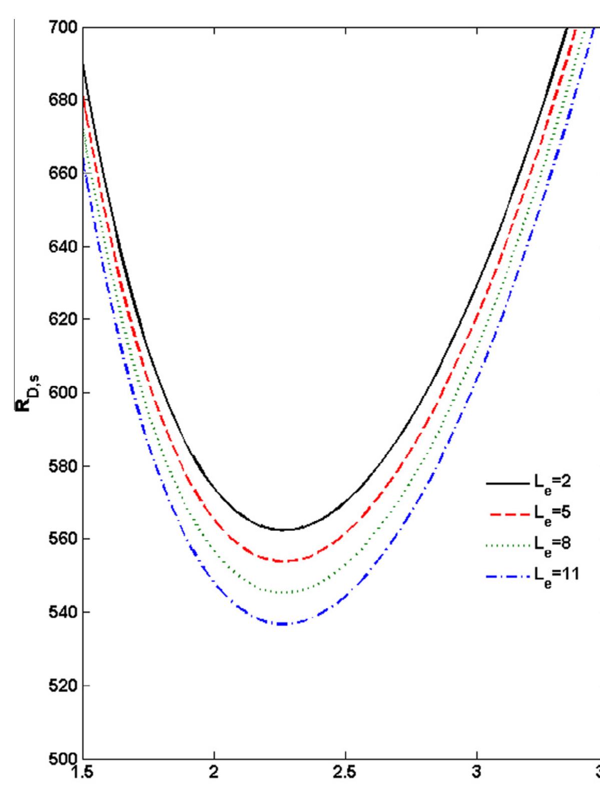

Figure 3 Variation of the Darcy–Rayleigh number $R_{D,s}$ with the wave number a for various values of the Lewis number L_e with $N_A = 2, \varepsilon = 0.7, D_a = 0.8, R_n = 1$.

and ω_i satisfies a dispersion relation of the form

$$F_1(\omega_i^2)^2 + F_2(\omega_i^2) + F_3 = 0, \tag{33}$$

where $F_1 = -D_a \varepsilon \alpha L_e^2$, $F_2 = \varepsilon\left[L_e^2[P_r + J\{D_a + (D_a + \alpha + D_a J\alpha)P_r\}] - D_a J^2 \alpha \sigma^2\right]$,

$$F_3 = \sigma\left[a^2 L_e N_A P_r R_n(\varepsilon + L_e - \sigma) + \varepsilon J^2 [P_r + J\{D_a + (D_a + \alpha + D_a J\alpha)P_r\}]\sigma\right] \quad \text{and} \quad J = a^2 + \pi^2.$$

On solving Eq. (33), the angular frequency ω_i of the oscillation is written as

$$\omega_i^2 = \frac{1}{2}\left[-\frac{F_2}{F_1} \pm \sqrt{\left(\frac{F_2}{F_1}\right)^2 - 4\frac{F_3}{F_1}}\right]. \tag{34}$$

Since ω_i is real for overstability, then the values of ω_i^2 should be positive. Hence, from Eq. (34), the necessary and sufficient conditions for occurrence of oscillation are

$$\left(\frac{F_2}{F_1}\right)^2 \geqslant 4\frac{F_3}{F_1}, -\frac{F_2}{F_1} \geqslant 0, \frac{F_3}{F_1} \geqslant 0. \tag{35}$$

Here $F_1 < 0$, then the necessary and sufficient condition for occurrence of oscillation is reduced to

$$F_2 \geqslant 0 \quad \text{and} \quad F_3 \leqslant 0. \tag{36}$$

Following Buongiorno [11] and Yadav [26] for most of nanofluids, the Lewis number L_e is on the order of $1 \sim 10^2$, N_A is on the order of $1 \sim 10$, the nanoparticle Darcy–Rayleigh number R_n and σ are on the order of $1 \sim 10$, and Hence from Eqs. (32) and (36), it is clear that oscillatory convection cannot occur for Kuvshiniski viscoelastic nanofluid and the principle of the exchange of stability is valid.

The results which are given in Eq. (30) are also presented graphically in Figs. 2–6 for fixed values of $N_A = 2$, $L_e = 5$, $\varepsilon = 0.7$, $D_a = 0.8$, $R_n = 1$ except the varying parameters. The range of parameters falling in these figures is taken from the available literature [26,33,52]. The linear stability theory expresses the criteria of stability in terms of the critical Darcy–Rayleigh number $R_{D,sc}$, below which the system is stable, while above which it is unstable.

Fig. 2 shows the effect of the concentration Darcy–Rayleigh number R_n on the stability of the system. It can be easily said that an increase in the values of the concentration Rayleigh Darcy number R_n leads to the decrease in the values of Darcy–Rayleigh number $R_{D,s}$, thus indicating an increase in the onset of convection. This may be understood as an increase in volumetric fraction and increases the Brownian motion of the nanoparticles, which causes the destabilizing effect of concentration Rayleigh Darcy number R_n.

The effect of Lewis number L_e and the modified diffusivity ratio N_A on the stability of the system is shown in Figs. 3 and 4, respectively. From these figures, we found that the Lewis number L_e and the modified diffusivity ratio N_A accelerate the onset of convection. It may be happened because the thermophoresis at a higher value of thermophoretic diffusivity is more supportable to the disturbance in nanofluids, while both thermophoresis and Brownian motion are driving forces in favor of the motion of nanoparticles.

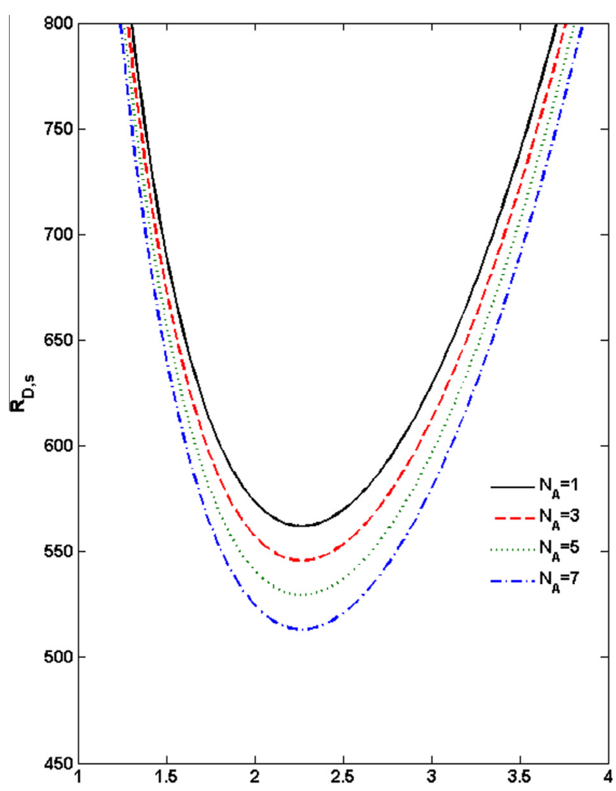

Figure 4 Variation of the Darcy–Rayleigh number $R_{D,s}$ with the wave number a for various values of the modified diffusivity ratio N_A with $L_e = 5, \varepsilon = 0.7, D_a = 0.8, R_n = 1$.

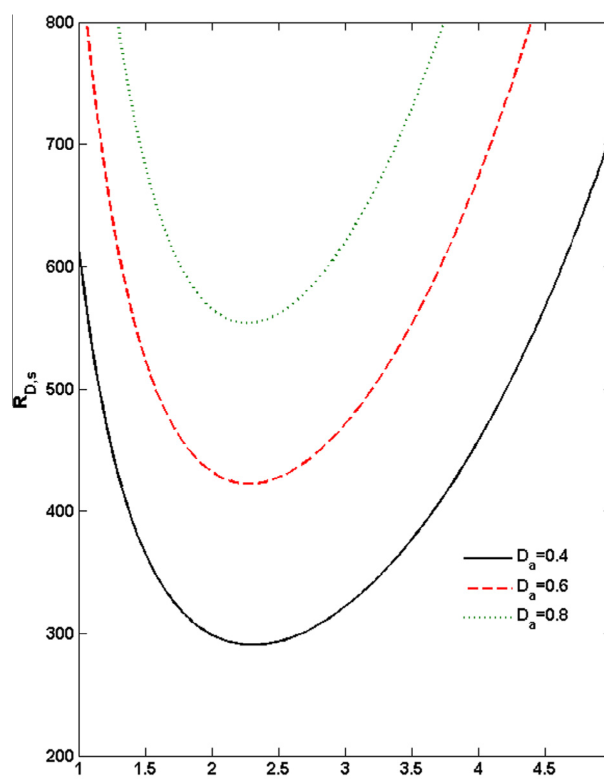

Figure 5 Variation of the Darcy–Rayleigh number $R_{D,s}$ with the wave number a for various values of the Darcy number D_a with $N_A = 2, L_e = 5, \varepsilon = 0.7, R_n = 1$.

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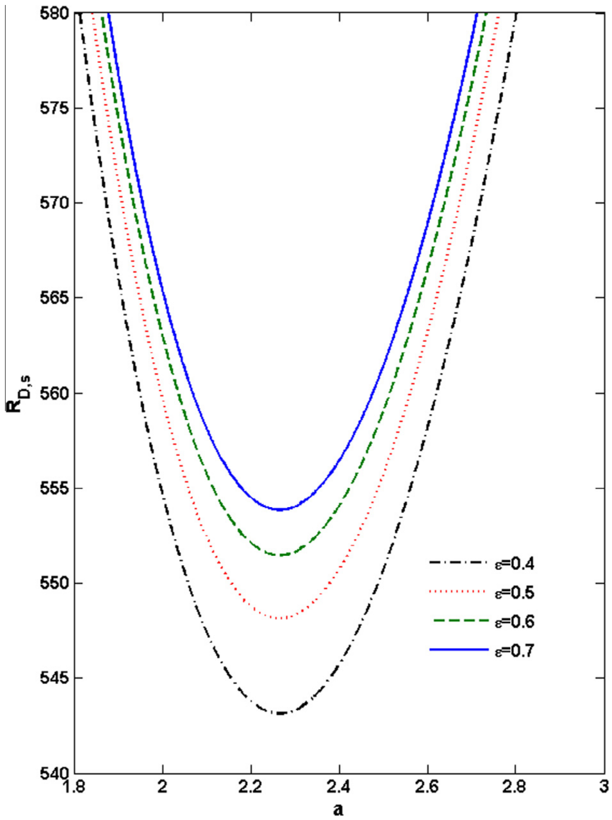

Figure 6 Variation of the Darcy–Rayleigh number $R_{D,s}$ with the wave number a for various values of the porosity parameter ε with $N_A = 2, L_e = 5, D_a = 0.8, R_n = 1$.

The effect of Darcy number D_a, on the natural curve is depicted in Fig. 5. The critical Darcy–Rayleigh number $R_{D,sc}$ increases with an increase in the Darcy number D_a which shows that the effect of Darcy number D_a delays the onset of convection in the nanofluid-saturated porous media, showing the stabilizing effect of the Darcy number D_a on the considered system.

Fig. 6 shows the variation of the critical Rayleigh number $R_{D,s}$ with the wave number a for different values of porosity of a viscoelastic nanofluid. From Fig. 6 it is observed that the critical Darcy–Rayleigh number $R_{D,sc}$ increases by increasing the porosity of a porous medium, showing the stabilizing effect of the porosity of a porous medium.

4. Conclusions

The thermal convection of Kuvshiniski viscoelastic nanofluid layer in a porous medium was analyzed analytically using the linear stability theory. The linear stability theory gives the condition for the onset of stationary convection and oscillatory convection. It is observed that the oscillatory instability is not possible for Kuvshiniski viscoelastic nanofluid and the principle of the exchange of stability is valid. The expression for the stationary convection shows that the Lewis number, the concentration Darcy–Rayleigh number and modified diffusivity ratio accelerate the onset of convection while the porosity parameter and Darcy number delay the onset of convection.

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